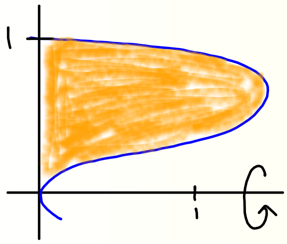
Calculus Warm Up # 6-3

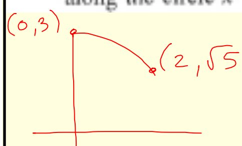
Use cylindrical shells to find the volume created when

$x = 12(y^2 - y^3)$ on [0, 1] is revolved about the x-axis.



HW Questions: p. 328

21. Find the arc length from (0, 3) clockwise to $(2, \sqrt{5})$ along the circle $x^2 + y^2 = 9$.



$y = \sqrt{9 - x^2}$

$y' = -\dfrac{x}{\sqrt{9 - x^2}}$

S

$(\quad)^2 = \dfrac{x^2}{9 - x^2}$

$S = \int_0^2 \sqrt{1 + \dfrac{x^2}{9 - x^2}}\, dx$

$\sqrt{\dfrac{9 - x^2 + x^2}{9 - x^2}}$

$= \int_0^2 \dfrac{3}{\sqrt{9 - x^2}}\, dx$

≈ 2.189

In Exercises 23–26, find the area of the surface of revolution generated by revolving the given plane curve about the x-axis.

23. $y = \dfrac{x^3}{3}$ $[0, 3]$

25. $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ $[1, 2]$

$$y' = \frac{1}{2}\left(x^2 - \frac{1}{x^2}\right)$$

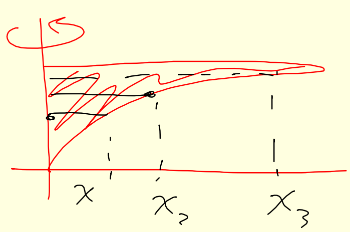
$$= \frac{1}{4}\left(x^4 - 2 + \frac{1}{x^4}\right)$$

$$S = 2\pi \int \left(\frac{x^3}{6} + \frac{1}{2x}\right)\sqrt{\frac{4 + x^4 - 2 + \frac{1}{x^4}}{4}}$$

$$= \frac{\pi}{2}\int \left(\frac{x^5}{3} + \frac{x}{3} + x + \frac{1}{x^3}\right) dx$$

In Exercises 27 and 28, find the area of the surface of revolution generated by revolving the given plane curve over the indicated interval about the y-axis.

27. $y = \sqrt[3]{x} + 2$ $[1, 8]$



$r = x$

$$y' = \frac{1}{3x^{2/3}}$$

Yellow review answers

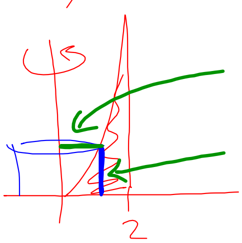
1) $\frac{256\pi}{15}$

2a) 16π b) $\frac{896\pi}{15}$ c) $\frac{16\pi}{3}$

3a) 8.1 b) $\frac{81\sqrt{3}}{40}$

2a)

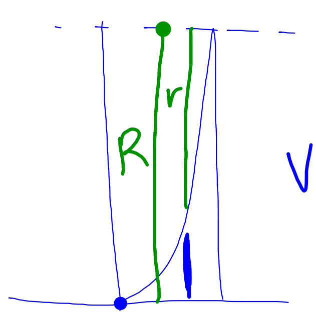
$$V = 2\pi \int_0^2 x(2x^2)\,dx$$



$r = x$

$h = 2x^2$

b)



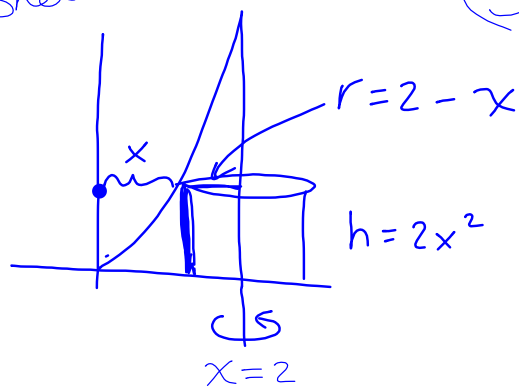
$y = 8$

$$V = \pi \int_0^2 (R^2 - r^2)\,dx$$

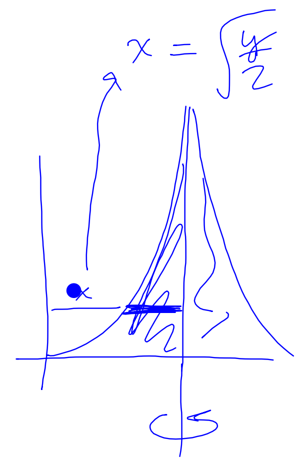
$R = 8$

$r = 8 - 2x^2$

2c) Shells

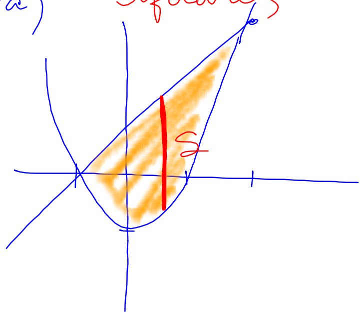


or discs:



$$V = 2\pi \int_0^2 (2-x)(2x^2)\,dx$$

3a) Squares



$$\int A(x)\,dx$$

$$s = x+1-(x^2-1)$$

$$s = -x^2+x+2$$

$$\int_{-1}^{2} (-x^2+x+2)^2\,dx$$

b) eq. Δ's $\longrightarrow$ $A = \frac{\sqrt{3}}{4} s^2$

$$\frac{\sqrt{3}}{4} \cdot \frac{81}{10}$$

$$=$$

6.5

Work done by a constant force

Work done by a variable force

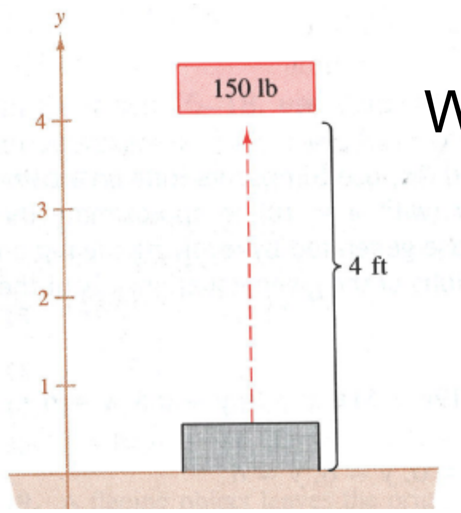
Work done by a constant force

W = Fd   F = a constant force
d = the distance the object is moved in the direction of the force.

*Notice: Work has only been done if the object moved!

No matter how hard I push on a wall, no work has been done if I didn't move the wall.

Determine the work done in lifting a 150-pound object 4 feet.



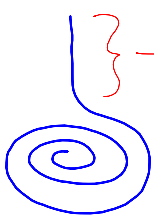
$$W = \overset{F}{(150 \text{ lbs})}\overset{d}{(4 \text{ ft})}$$

$$= 600 \text{ ft-lbs.}$$

Work done by a variable force

F(x) : the amount of force needed to move the object changes as the object's position, x, changes.

Example: Lifting a heavy chain

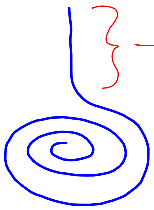


→ the length of the chain you are supporting as you lift increases.

☆ It gets heavier.

The chain weighs 5 lbs/ft.

Let Δy = incremental length of chain being lifted



y = total distance

$$F(y) = \frac{5\text{ lbs}}{\text{ft}} \cdot \Delta y\text{ ft}$$

$$= 5\,\Delta y\text{ lbs}$$

$$W = Fd \qquad W = \int_0^h 5\,y\,dy$$

Force

distance

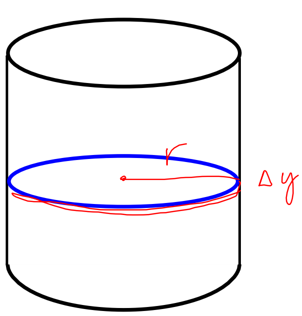
(h = height you want to lift it to)

Work done by a variable force

If an object is moved along a straight line by a continuously varying force $F(x)$, then the **work** W done by the force as the object is moved from $x = a$ to $x = b$ is given by

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta W_i = \int_a^b F(x)\,dx.$$

Work required to move liquid:

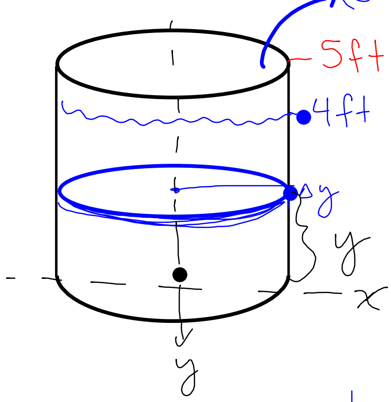


Force needed to move the liquid varies with the volume and weight of the liquid.

$$F = \left(\text{weight per unit of volume}\right)\left(\text{volume of incremental "layer"}\right)$$

$r = 2$ ft

distance $(5-y)$ ft



water weighs $\rightarrow \dfrac{62.4 \text{ lbs}}{ft^3}$

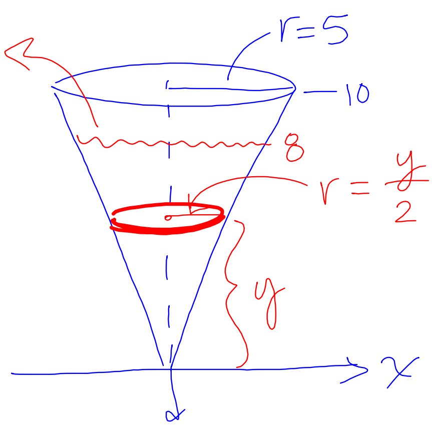
$$F = \frac{62.4\text{lbs}}{\cancel{ft^3}} \left(\pi (2)^2 \Delta y\right) \cancel{ft^3}$$

$$F = 249.6\pi (\Delta y) \text{ lbs}$$

Work done to remove all water out top.

$$W = \int_0^4 (5-y)(249.6\pi)\, dy$$

limits of integration are where the water is in the tank.



weight of oil $= \dfrac{57 \text{ lbs}}{ft^3}$

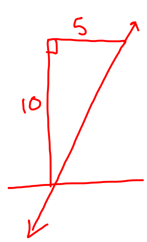
$$F = \frac{57\text{lbs}}{\cancel{ft^3}} \cdot \pi\left(\frac{y}{2}\right)^2 \Delta y \cancel{ft^3}$$

$$F = \frac{57\pi y^2}{4} \Delta y \text{ lbs}$$

$$W = \int_0^8 (10-y)\left(\frac{57\pi y^2}{4}\right) dy$$

$W = Fd$

$d = (10-y)$ ft



$y = 2x$

$x = \dfrac{y}{2}$

HW: p. 335

1 - 3, 11-15, 17

Individual Quiz: Thursday

Volume, 6.2 - 6.3